Routing Models for Rural Networks with Time-Varying Constraints

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Abstract

The motivation for this work comes from the poultry industry, but can be broadened to other application areas. One problem of concern in the poultry industry is when an infected flock of birds has to be transported to another facility, but in doing so, the infected flock cannot come within a certain radius (e.g., five miles) of a breeding (or some other type of) facility. Alternately, a feed truck may not be allowed to come within a certain radius of an infected site/area. The poultry industry often has trouble solving these types of problems. In fact, they tend to solve the most restricted form of this problem, assuming a static radius value over time, rather than the real problem where the radius might vary over time, depending on site-specific conditions. Furthermore, conditions are clearly variable, as the disease may be one that spreads in the air and the spreading mechanism is dependent on the passage of time, wind speed, and other stochastic factors. The network that connects poultry facilities is primarily rural.

To address this problem, we took a systems perspective and developed a method that integrates the Geo-Spatial information available on rural transportation routes with logistics decision support knowledge tools to efficiently route the movement of infected flocks while minimizing the risk of exposure to other poultry farms. In this report we present a mathematical model for executing pickups and deliveries in a rural network with a fleet of capacitated vehicles under time constraints. We demonstrate the use of this model in transporting live chickens in a rural poultry network in Northwest Arkansas. The data for this network was obtained through a Geographical Information Systems (GIS) database. This research also applies to the transportation of toxic waste, network routing where rush hour traffic is a concern and other important transportation applications where the network changes over time in a stochastic manner

1 Introduction

Diseases and infection within the poultry industry pose a serious threat as they can be transmitted to other poultry and can lead to food contamination. To counter this threat, various emergency response procedures are in place. An outbreak of an infectious disease in poultry is communicated to the parties involved such as farms, hatcheries, breeding facilities and kill facilities as well as any party that uses any roads that are within the outbreak zone. The infected area is placed in a quarantine zone and the movement of poultry within this zone is restricted. Due to these restrictions, transporters of poultry are forced to find alternate routes to reach their destinations. In this work, we consider only the scenario where complete information about an outbreak is known prior to solving the problem.

A description of the underlying problem is as follows: vehicles are located initially at their base depots. Facilities (farms, hatcheries, etc.) located at geographically-dispersed locations place requests for the transportation of birds. A request consists of transporting the birds from their current location to another facility (kill facility, breeding facility, etc.) within a stipulated time window. These requests are placed on a daily basis. The vehicles return to a pre-specified destination depot after delivering the birds to their delivery point. Based on the definition of [5], this problem can be characterized as a general pickup and delivery vehicle routing problem with time windows.

Cordeau *et al.* [5] present a review of pickup and delivery vehicle routing problems and describe the heuristic and optimization-based approaches that have been proposed to solve such problems. Sigurd *et al.* [2] investigate the problem of scheduling the routing of live animals according to non-decreasing order of their health levels. They study the problem as a vehicle routing problem with time windows and precedence constraints. Over the last couple of decades, dynamic pick and delivery type vehicle routing problems with time windows have been widely studied and numerous solution approaches have been proposed. Desrosiers *et al.* [4] presents a comprehensive overview of dynamic vehicle routing problems.

The rest of the paper is organized as follows. Section 2.1 describes a pickup and delivery model considering capacity and time constraints based primarily on the model presented in [5]. Section 2.2 discusses a solution methodology applicable for the scenario where complete information about an outbreak is known prior to solving the problem. Section 3 demonstrates the use of our model on an actual dataset from the poultry industry and Section 4 gives concluding remarks and identifies areas for future research.

2 Model Description

2.1 Mathematical Model

In this work we make the following assumptions:

- Each vehicle has a finite capacity,
- the total capacity of all the vehicles is sufficient to transport all loads,
- each load has a single destination, and
- individual items must be delivered at the delivery node corresponding to where it was picked up.

The notation used in our model is found below:

Sets:

K	:=	Set of vehicles indexed by k
P	:=	{1,...,p}: Set of pickup locations (P_k for vehicle k only)
D	:=	$\{p+1,\ldots,2p\}$: Set of delivery locations D_k (for vehicle k only)
$o(k)$	:=	origin depot of vehicle $k \in K$
$d(k)$	:=	destination depot of vehicle $k \in K$
N_k	:=	$P_k \cup D_k$
A_k	:=	$\{(i,j) : i \in N_k, j \neq i - p \in N_k\} \cup \{(o(k),j) :$ $j \in P_k \cup d(k)\} \cup \{(j,d(k)) : j \in D_k\}$

Parameters:

c_{ij}^k	:=	Cost of using arc $(i,j) \in A_k$ by vehicle $k \in K$
l_i	:=	Load to be picked up at location $i \in P$ and to be delivered at location $p+i \in D$
Q_k	:=	Capacity of vehicle k
t_{ij}^k	:=	Time required for vehicle $k \in K$ to travel arc $(i,j) \in A_k$
s_i	:=	Service time for node i
$[a_i, b_i]$	:=	time window for node i

Variables:

$$x_{ij}^k = \begin{cases} 1, & \text{if vehicle } k \in K \text{ uses arc } (i,j) \in A_k; \\ 0, & \text{otherwise.} \end{cases}$$

L_{id}^k	=	Load to be delivered to location d on vehicle $k \in K$ after visiting location $i \in V_k$
T_i^k	=	Time when vehicle $k \in K$ starts service at location $i \in V_k$

In our pickup and delivery model, there are a set of vehicles K located at their origin depots $o(k)$. Request for transportation of birds is placed at several pick-up locations numbered $1,\ldots,p$. Each load is assumed to have a single destination, where each destination is numbered $p+1,\ldots,2p$. Since there are p loads to be picked up, it is convenient to have the same number of destinations represented mathematically although two or more destinations may be the same physical location. After completing its route, each vehicle returns to its destination depot $d(k)$. For the purpose of tractability, we do not allow vehicles to visit the same node twice in our model (a.k.a. backtracking). It is important to note that A_k is defined in a way to the eliminate the binary decision variables that are infeasible based on our assumptions (delivery to pickup, destination to origin, pickup to destination, and pickup or delivery to origin arcs) and is based on [1].

We formulate our model as a mixed-integer program (MIP) in (1–13):

$$\min \sum_{k \in K} \sum_{(i,j) \in A_k} c_{ij}^k x_{ij}^k \tag{1}$$

$$
\text{s.t.} \quad \sum_{k \in K} \sum_{j \in N_k \cup \{d(k)\}} x_{ij}^k = 1, \qquad \forall i \in P \tag{2}
$$

$$
\sum_{j \in N_k} x_{ij}^k - \sum_{j \in N_k} x_{j,p+i}^k = 0, \qquad \forall k \in K \tag{3}
$$

$$
\sum_{j \in P_k \cup \{d(k)\}} x_{o(k),j}^k = 1, \qquad \forall k \in K \tag{4}
$$

$$
\sum_{i \in N_k \cup \{o(k)\}} x_{ij}^k - \sum_{i' \in N_k \cup \{d(k)\}} x_{ji'}^k = 0, \qquad \forall k \in K, \forall j \in V_k \tag{5}
$$

$$
\sum_{i \in D_k \cup \{o(k)\}} x_{i,d(k)}^k = 1, \qquad \forall k \in K \tag{6}
$$

$$
T_i^k + t_{ij}^k \leq M(1 - x_{ij}^k) + T_j^k, \qquad \forall k \in K, \forall (i,j) \in A_k \tag{7}
$$

$$
a_i \leq T_{ik} \leq b_i, \qquad \forall k \in K, \forall i \in V_k \tag{8}
$$

$$
T_{ik} + t_{i,p+i,k} \leq T_{p+i}, \qquad \forall k \in K, \forall i \in P_k \tag{9}
$$

$$
L_{i,d}^k + l_j \leq M(1 - x_{ij}^k) + L_{j,d}^k, \qquad \forall k \in K, d \in D, (i,j) \in A_k \tag{10}
$$

$$
0 \leq L_{p+i,p+i}^k \leq Q_k - l_i, \qquad \forall k \in K, \forall p+i \in D_k \tag{11}
$$

$$
l_i \leq \sum_{d \in D} L_{i,d}^k \leq Q_k, \qquad \forall k \in K, \forall i \in P_k \tag{12}
$$

$$
x_{ij}^k \in \{0,1\}, \qquad \forall k \in K, \forall (i,j) \in A_k \tag{13}
$$

The objective, (1), is to minimize the total transportation cost incurred by the entire fleet of vehicles. In (2), every pickup node is constrained to be visited by exactly one vehicle and in (3), that vehicle also must visit the corresponding delivery node. Constraints (4)–(6) are standard in multicommodity flow models and ensure that the route for each vehicle begins at its origin $o(k)$ and terminates at its destination $d(k)$. Constraint (7) keeps track of the time that a vehicle visits each node and (8) enforces time windows. In (9), a pickup must be visited before its delivery. Constraint (10) updates the load quantity at each node, (11) decrements the load when it makes a delivery, and (12) enforces capacity requirements. Finally, (13) requires that the main decision variable be binary.

2.2 Solution Methodology

A slight variation exists in this problem context in the case where a disease outbreak has occured prior to solving the problem instance and therefore movement of poultry within a defined quarantine zone is either not allowed or not preferable for the entire planning period. In this case, we either remove each of the arcs in the quarantine zone or add arbitrary values to each of their weights. We then use the updated information about the network to obtain the shortest paths between all pairs $(i, j) \in A_k$ for all k, which are needed to solve the model defined in (1–13). This step will also help the decision maker determine if any node $i \in P \cup D$ is “unreachable.” In addition, the model presented in (1–13) can also be used for decision making when a disease outbreak occurs during the execution of a routing plan. The steps are as follows:

1. Remove arcs in quarantine zone
2. Move vehicles in quarantine zone to nearest "safe" node

3. Change origins of vehicles
4. Remove nodes corresponding to completed deliveries
5. Remove all unecessary arcs
6. Set initial loads to amounts carried at time of outbreak
7. Suppose for a particular vehicle, all farms that delivery to a single hatchery have been visited and the hatchery has not been visited. Add a constraint specifying that the hatchery must be visited by that vehicle.
8. Obtain all pairs shortest path for reduced network
9. Branch and bound to solve model
10. Translate solution into a routing plan

3 Model Demonstration

We demonstrate our model by solving a pickup-and-delivery problem on a poultry network in Northwest Arkanasas. The numeric data for this network was generated using GIS (Graphical Information System) data provided by the Center for Spatial Technologies (CAST) at the University of Arkansas. In particular, this example has 1 vehicle, 12 farms, and 2 hatcheries and is shown in Figure 1. The origin and destination for the vehicle was chosen arbitrarily and the demand for each of the farms was chosen randomly from the interval $[1, 10]$. We assume that the route starts at 8a and the vehicle travels 60 miles per hour. Due to the fact that GIS road network for the area of Northwest Arkansas that contains this set of 12 farms and 2 hatcheries includes about 25,000 roads or road sections (each of which would correspond to x_{ij}^k decision variables in our model), we elected to filter the data to only include roads or road sections that are either (1) in close proximity to a farm or a hatchery or (2) have a high amount of traffic (defined by the *roadClass* in the GIS data). While this does eliminate possible solutions, it improves tractability while keeping the network fully connected.

We first solve the filtered dataset without time windows or vehicle capacity using CPLEX 10.1 with a 3.0 GHz processor and 4GB of RAM. The optimal solution was obtained in less than 1 second and the optimal cost (in miles) was 90.7. The optimal solution is shown in Figure 2(a). In the optimal solution we observe that the maximum load carried by the vehicle is 64 demand units. To demonstrate the capacity functionality of our model, we restrict the capacity of the vehicle to 63 units and resolve the model. The optimal solution was obtained in 8 seconds and the optimal cost (in miles) was 98.2. The optimal solution is shown in Figure 2(b).

In the solution to our capacitated model, we observe that hatchery 2 is visited at 10a. To demonstrate the time window functionality of our model, we constrain that hatchery 2 must be visited before 9:55a and resolve the model. The optimal solution was obtained in 1.7 hours and the optimal cost (in miles) was 120.8. The optimal solution is shown in Figure 3.

As mentioned in section 2.2, our model can be used as a planning tool in the aftermath of a disease outbreak within the poultry distribution network. To demonstrate this, we examine two scenarios. In the first scenario, an outbreak has occurred prior to the solving of the model. The consequences of this outbreak are that the first arc used in the previous solution cannot be used (quarantined). To account for this, we add a large number to the cost of the first arc used in the previous opimal solution. The optimal solution to this model was obtained in 3.3 hours and the optimal cost (in miles) was 125.8. The optimal solution is shown in Figure 3.

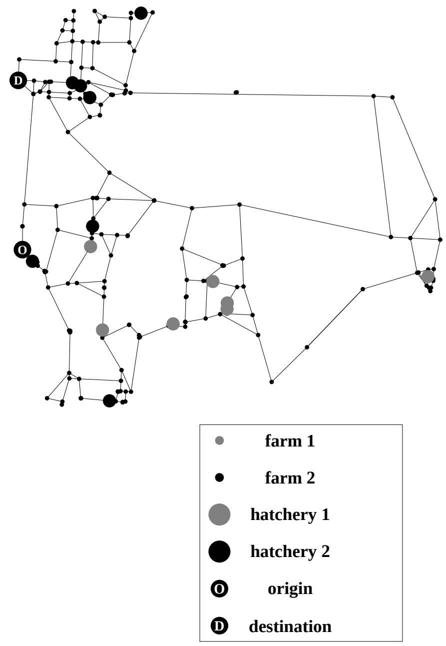
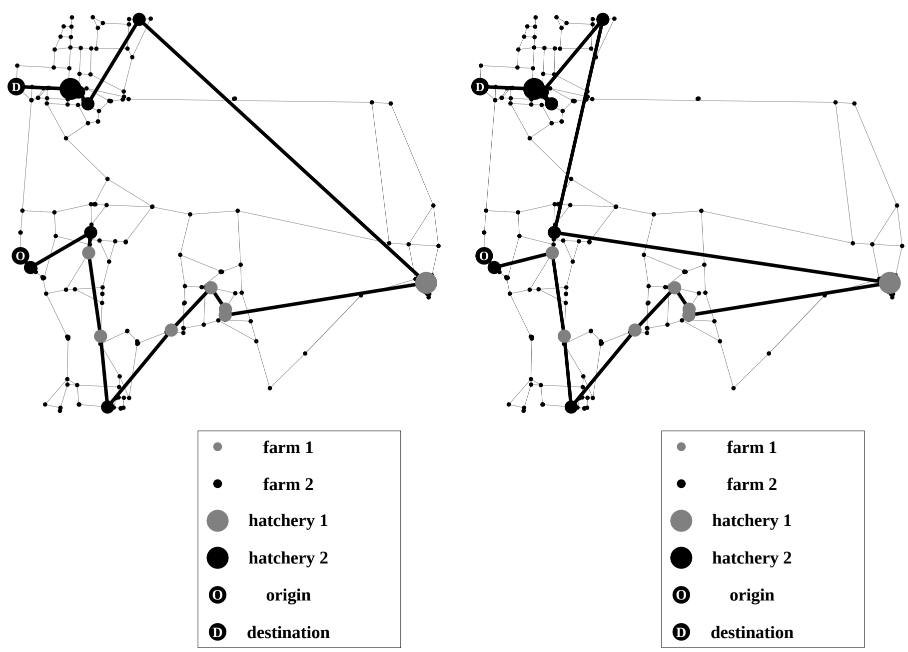


Figure 1: Network of poultry farms in Northwest Arkansas.



(a) uncapacitated

(b) capacitated

Figure 2: Solutions for uncapacitated and capacitated datasets.

In the second scenario an outbreak occurs at 8:39a, during execution and shortly after the 7th node is visited. A consequence of this outbreak is that the 7th arc used in the previous optimal solution cannot be used. In keeping with the instructions given in section 2.2, we remove the farms visited before the outbreak from the dataset and remove all arcs that are

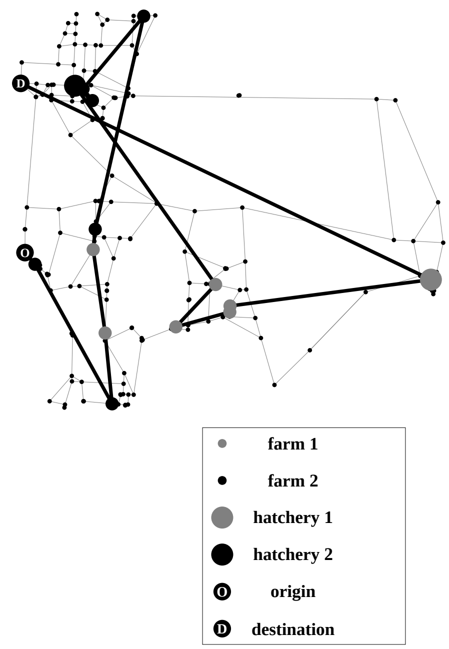


Figure 3: Solution for capacitated dataset with time windows.

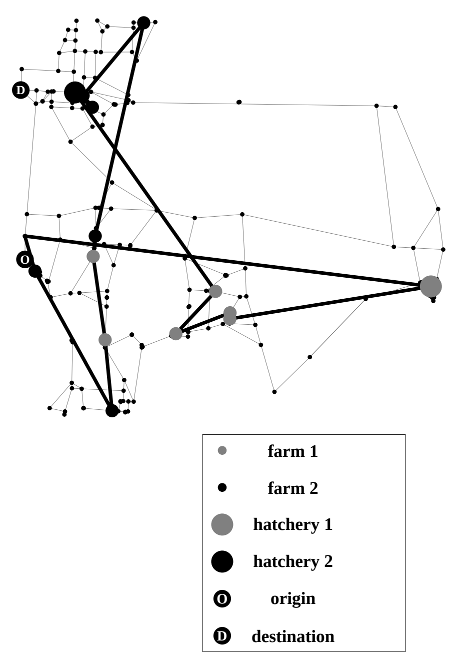


Figure 4: Solution for dataset with time windows with a disruption that occurs prior to solving.

connected to them. Because 7th node was visited most recently (and is closest in distance) we select it as the origin in our new model. Further, we remove all arcs traveling into the 7th node, because they are no longer necessary. Finally, we add a large number to the arc cost of the 7th arc visited in the previous optimal solution. The optimal solution to this model was obtained in less than 1 second and the cost (in miles) for the optimal path visiting each of the remaining nodes in the network was 81.7. The optimal solution is shown in Figure 5 with the optimal path before the outbreak shown in gray and the path after the outbreak shown in black.

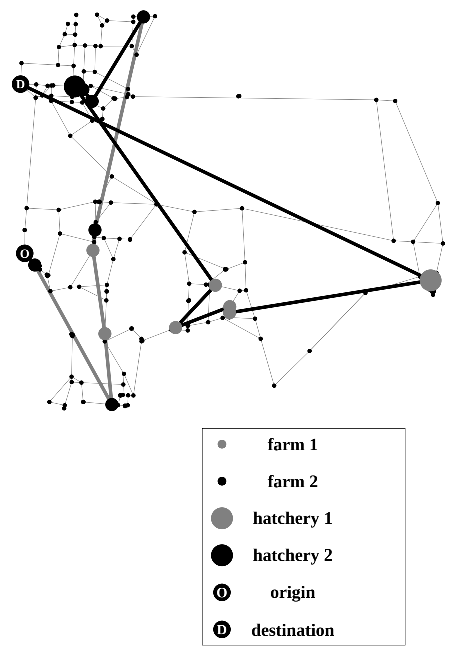


Figure 5: Solution for dataset with time windows with a disruption that occurs during route execution.

As Figure 5 shows, the optimal solution itself is not sufficient for route planning because it simply gives the sequence of farms and hatcheries to visit rather than which roads to use. Thus, the optimal paths obtained between each of the nodes visited on the optimal path must be obtained. The optimal solution with optimal intermediate paths is shown in Figure 6.

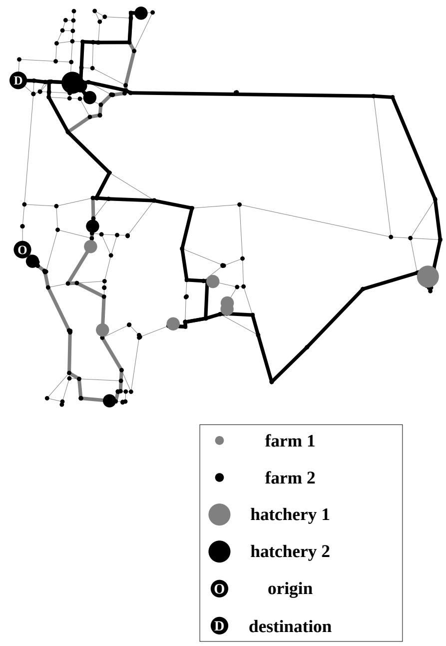


Figure 6: Translated solution for dataset with time windows with a disruption that occurs during route execution.

4 Conclusions and Future Research

We presented a mathematical model for executing pickups and deliveries in a rural network with a fleet of capacitated vehicles. In addition, we gave a method to address the scenario that an outbreak occurs prior to solving the problem and complete information about the outbreak is known. Finally, we demonstrated the validity of our model by solving datasets with and without time windows.

As a first item of future research, we plan to investigate relaxing some of our assumptions stated in Section 2. Thus, we plan on formulating models to accommodate backtracking or non-homogeneous loads, or both. Relaxing the backtracking assumption can improve the objective function value and including non-homogeneous loads in our model is of use in this problem domain when it is desired that certain poultry loads not be delivered to certain hatcheries or certain types of poultry not be transported together. In addition, we plan on using heuristics to solve this problem for the purpose of comparison with our model and solving larger instances. Finally, we plan to attempt to find valid inequalities that are particularly useful in our problem context.

Another important area that we did not consider in this work is the dynamic spread of disease within a population of poultry. Thus, as a future item of research, we plan to work with experts in the field of poultry science to develop disease spreading models.

There exist several other problem classes that we did not consider in this work that are relevant to real-life operations in the poultry industry. One such problem class is what we call the *deterministic rolling horizon problem*. This problem class is dynamic in that the network is changing over time and is characterized by the deterministic spread of disease. An extension of the deterministic rolling horizon problem is the *stochastic rolling horizon problem*. In this problem class the disease spreads in a stochastic, rather than a deterministic, manner. The final problem class of interest is the *planning under risk problem*. Problems in this class deal with constructing vehicle route plans given that the probability of an outbreak is known for each of the farms.

Acknowledgements

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